# High School
# Course of Study Series

PART TWO
SECTION IV

# MATHEMATICS

*Published by*
DEPARTMENT OF PUBLIC INSTRUCTION
TERRITORY OF HAWAII
*September* 1, 1927

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## Sections Included in Part Two

## of

# High School Course of Study Series

**Section I. COMMERCIAL**

(Prepared in Mimeograph form only)

Advertising—Bookkeeping—Business English—Commercial Law—Elementary Business Training—Junior Business Training—Office Machine Practice—Office Training—Salesmanship—Shorthand—Typing.

**Section II. ENGLISH**

English—English (1 and 2)—English (3 and 4)—English (5 and 6)—English (7 and 8)—English Reading Lists—Argumentation—Dramatics—Journalism—Library Training—Public Speaking.

**Section III. FOREIGN LANGUAGES**

Chinese—French—Hawaiian—Japanese—Korean—Latin—Spanish.

**Section IV. MATHEMATICS**

Algebra, Elementary—Algebra, Advanced—Business Arithmetic—Geometry, Plane—Geometry, Solid—High School Mathematics (Special)—Trigonometry.

**Section V. SCIENCE**

Agriculture—Astronomy—Biology—Chemistry—Geology—General Science (2 Semesters)—Elementary Science (1 Semester)—Physics.

**Section VI. SOCIAL SCIENCE**

American Problems—Ancient and Medieval History—Civics, General—Elementary Civics and Occupational Civics—Commercial Geography—Economics—Hawaiian History—History of Industry—Modern History—Oriental History—United States History—World History.

**Section VII. HOUSEHOLD ECONOMICS**

(Prepared in Mimeograph form only)

Cafeteria Management—Cooking (Elementary and Advanced)—Home Management—Sewing (Elementary and Advanced).

**Note:** For Vocational Home Making see another pamphlet published by the Department of Public Instruction.

**Section VIII. MANUAL AND VOCATIONAL EDUCATION**

(Prepared in Mimeograph form only)

Agriculture—Auto Mechanics—Metal Working—Mechanical Drawing—Wood Working.

**Section IX. MISCELLANEOUS**

Graphic Art—Freehand Drawing—Design—Commercial Art—Hawaiian Arts—Music—Physical Education.

# INTRODUCTION

Feeling the need for a more complete standardization of high school work, a tentative Course of Study was drawn up in 1924 with the cooperation of the high school principals. This Course of Study was put out in mimeographed form for experimental use in all high schools of the Territory. As the result of invited criticisms and suggestions, a revision was mimeographed in 1925. It was then expected that the Course of Study would be printed in 1926. The "Teachers' Manual" was published September 1, 1926, as Part I of the High School Course of Study Series. The separate courses of study themselves were delayed until 1927, so that more changes might be incorporated.

Although this Course of Study has gone through three revisions, it is still far from complete, because changes in both theory and practice must be made continually if any school system is to keep abreast of the needs of its students. This series is, however, offered now in printed form, so that it can have a wider distribution. It is hoped that every teacher will take a keen interest in interpreting the subject matter of the classroom to fit the individual needs of the students, and that this Course of Study outline will be suggestive and helpful in leading the way.

The separate courses of study have been worked out entirely by the schools themselves, through the splendid cooperation and loyal effort of both principals and teachers. Special thanks are due the High School Course of Study Committee, which has compiled and revised the completed series. The committee is as follows:

### General Chairman

Mr. Miles E. Cary, Principal of McKinley High School.

### Central Committee

Mr. J. P. Buller, Vice-Principal, McKinley High School.
Mr. James R. Coxen, Territorial Director of Vocational Education.
Mrs. Lura J. Loader, Teacher of English at McKinley High School.
Mr. Oren E. Long, Deputy Superintendent of Public Instruction.
Mr. T. M. Livesay, Professor of Education, University of Hawaii.
Mr. Robert R. Spencer, Principal of Washington Junior High School.
Dr. Ross B. Wiley, Director of Research, Department of Public Instruction.
Mr. Benj. O. Wist, President of the Territorial Normal School.

### General Committee

Mr. F. A. Clowes, Principal Leilehua High School.
Mr. Clyde E. Crawford, Principal, Konawaena High School.
Mr. F. E. Howard, Principal, Maui High School.
Mr. W. Harold Loper, Principal, Kauai High School.
Mr. Richard E. Meyer, Principal, Hilo High School.
Mr. H. Alton Rogers, Principal, Lahainaluna High School.
Mr. Harlan M. Roberts, Principal, Kohala High School.

WILL C. CRAWFORD,
Superintendent of Public Instruction.

# High School Course of Study Series

# Part Two

## Section IV—Mathematics

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# High School Course of Study Series

## PART TWO

## GENERAL STATEMENT

The central aims of secondary education have been outlined in Part One of the high school course of study. An attempt will be made in the following outlines to establish a basis for the actual classroom procedure in accordance with those aims.

Those persons who have been closely associated with the work of the various committees will realize that it has been a tremendous task to harmonize the work of some seventeen committees where the members of all committees (except the Central Committee) were scattered throughout the islands.

## THE GENERAL PLAN

In order that all committees might have a common point of departure in their work the following basic directions were drawn up by the Central Committee:

1. Be specific in your statements; generalizations often lead afield.
2. Secondary education is more concerned with the activities of the individual than it is with making the student's mind a store house of information.
3. In so far as possible every teacher and every recitation is concerned with the three elements of social efficiency, namely: efficiency of government, efficiency of industry, and efficiency of home. In other words, our various subjects are not so many airtight compartments each concerned only with a narrow range of purposes. Each teacher who works on the course of study should endeavor to see that through the pupil activities, provided for in each subject, definite growth will be realized in the direction of the **socially efficient** individual.
4. Examine all of your contributions in the light of the major aims of education as outlined in Part One of the Teachers' Manual.
5. A minute arrangement of the subject matter by weeks and months is not wanted. Every teacher, worthy of the name, is capable of doing this. But what is wanted is a general outline of the basic amount of work to be covered by each subject.
6. It is expected that slower pupils will not be able to cover the basic amount of work outlined in each subject. Modifications in subject matter will need to be made for slow students.

The following quotation, taken from the National Educational Association research bulletin entitled, "Keeping Pace with the Advancing Curriculum," was adopted by the Central Committee in order to establish the dominant tone of these outlines:

> "The teacher is a trained observer who guides the interests of her pupils into activities that are purposeful and which have social value. In the classroom, the subject-matter is still subordinate to activities. Whereas the teacher has clearly in mind a unified and carefully organized core of subject-matter, she does not rigidly require that it shall be covered in any set order."

The course of study outlines which follow represent the work and interest of many persons. These outlines have been printed in pamphlet form for the convenience of teachers. However, this does not mean that the course of study work is complete. This work can

never be complete, but must continuously give due consideration to the needs of our ever changing society. However, with these outlines as a point of departure, each teacher is urged to carry forward the work; and the real test of the value of these outlines will be the extent to which they are used.

It should be clearly understood by all that these outlines are a guide, rather than a hitching post.

## SCHOOL ECONOMY

**CLASSROOM ECONOMY:** This is an essential factor in school work. Every minute wasted is a loss of as many minutes as there are students in the class. This may amount to many hours, even days or weeks for the school year. Efficiency is the watchword in both business and education. Classroom economy demands:

1. **Beginning work at the ringing of the last bell—or before if the class is assembled.**
2. **Seating of students so that roll may be called by inspection in a fraction of a minute.**
3. **Good order at all times—loud talking and boisterous conduct should not be allowed even between classes.**
4. **Attention to the work in hand—reading papers, preparing lessons for other classes, and sitting listlessly should not be tolerated.**
5. **That assignments be definite and clear, and at the beginning of the recitation period; unless the principles under discussion lead logically to new principles to be assigned for study later.**
6. **System in passing out papers, laboratory material, books and supplies.**
7. **Proper ventilation and adjustment of shades to regulate the light.**
8. **Cheerfulness at all times and occasional praise for excellence as incentives for good work.**
9. **Prompt dismissal at the ringing of the first bell.**

**STUDY-HALL ECONOMY:** The following rules should be firmly enforced:

1. Independent work on the part of every student.
2. Order—no whispering or talking.
3. Punctuality.
4. Industry.
5. System in roll call.
6. System in dismissing students to go to the library.

**HALL ECONOMY:** A great deal of time is wasted in the halls and corridors. Tardiness is frequently due to this fact. This waste may be reduced to minimum if students are urged:

1. To walk lively, but orderly.
2. To follow the rule keeping to the right, or left, as the case may be.
3. To take the shortest route.
4. To refrain from whistling and boisterousness at all times.
5. To take pride in good order and promptness to class.

During school hours the school is a work shop. If teachers are able to establish in the school the same spirit of work and application which one expects to find in a work shop, then the school will be approaching the ideal in the matter of working efficiency.

## SOME FUNDAMENTAL CONSIDERATIONS

It should be the desire of each high school teacher to give every student, who comes under his instruction, a training commensurate with the capacity of that student.

This is also the aim of the Department of Public Instruction. The general aims of secondary education have been outlined in Part One of this series.

The primary purpose of Part Two is to outline in a general way the aims and content of each subject offered.

This procedure seems to be necessary in order to coordinate the work of the various secondary schools of the Territory.

But in addition to outlining the formal work of each class, it may also be necessary to add a suggestion or two regarding the place which the formal matter should play in the class work:

1. The subject matter of any course should be considered as a means to an end, rather than the end.
2. If the subject matter of a course is to be used as a tool, then the student should know how to use this tool, or tools. In general the student should be led to use his facts and skills in the solution of problems. The problems, or projects, should be those which have an appeal to the student; better, they should be the student's problems. If, as has just been stated, subject-matter is a tool, it must be a tool to some end. It must serve some purpose. For the teacher to be conscious of this purpose is not enough. The pupil must also be conscious of it. Otherwise that which is intended to serve him as a tool is mere useless rubbish to him. Furthermore, it should be borne in mind in this connection that purpose must exist **before** tools are needed,—**not after.** The teacher may assist the student in discovering worth while and interesting problems. But the student's response is apt to be poor if the teacher arbitrarily assigns problems which interest her, regardless of the interests of the students. As experience amply proves, it is almost impossible to interest every student in a class. But if each teacher will keep in mind that interest generally precedes learning, then a great deal of waste of human energy may be avoided—for the teacher as well as the pupil.

When the teacher faces her class at the beginning of a term, she is not prepared to meet the full responsibility and opportunity of the occasion unless she can answer the following questions, without qualifications, in the affirmative:

1. Am I prepared to teach this subject?
2. Do I see each student as an individual, eager for all that I can give of my store of knowledge and inspiration?
3. Do I consider character training to be of more importance than the giving of information?
4. Knowing that boys and girls learn largely by imitation and emulation, am I willing to order my own conduct, both in school and out, in harmony with the ideal for which I am expecting the students to strive?
5. Am I willing to be a personal friend of each student in my class?
6. Am I willing to give more than is outlined in the textbook and manual?

If each teacher can answer these questions in the affirmative, then the high schools will prosper, and each student will go forth well equipped to meet the tremendous and complex demands of modern society.

# MATHEMATICS

## GENERAL AIMS OF MATHEMATICS:

1. **To give an understanding of the common terms, symbols, and concepts used by the world in its quantitative thinking.**
2. **To prepare certain students to meet the entrance requirements of colleges and universities.**
3. **To provide special training for those students who desire to specialize in the study of mathematics, both in high school and in college.**

(Adapted from Bureau of Education Bulletin, 1920, No. 1—The Problem of Mathematics in Secondary School.)

**Note:** On account of the uncertainty regarding the amount of mental discipline which the study of mathematics provides, and just how this discipline is best promoted, no aim has been included in the foregoing list, based upon this time-honored purpose of mathematical instruction. However, until these questions are scientifically settled, mathematics should be taught thoroughly, wherever and whenever it is taught. Students will certainly acquire habits of thoroughness and persistence, or the opposite, just as readily in a mathematics classroom as in study of any other subject.

## SUBJECTS INCLUDED UNDER MATHEMATICS:

Algebra, Elementary.
Algebra, Advanced.
Business Arithmetic.
Geometry, Plane.
Geometry, Solid.
High School Mathematics, Special.
Trigonometry.

# ALGEBRA—ELEMENTARY

## AIMS OF ELEMENTARY ALGEBRA:

1. To define algebra in the mind of the pupil as a useful, interesting and rational subject.
2. To teach the solution of simple linear equations by means of a series of logical steps.
3. To lay the foundation for a sound method of problem solution.
4. To enlarge in the mind of the pupil the meaning of the minus sign as something more than a sign of operation.
5. To achieve mastery in the fundamental operations on signed numbers.
6. To develop power in the solution of equations and to make algebra a practical, cultural subject rather than a mechanical drill. To strengthen the pupil's growing power to write the shorthand of algebra and to translate it into words.
7. To enlarge the pupil's control over multiplication and division.
8. To teach practical use of factoring in quadratic equations and problems.
9. To give drill enough in fractions to make use of fractions in fractional equations possible. To develop the idea of function by ratio and proportion.
10. To give graphic form the relation of quantities.
11. To enlarge the pupil's acquaintance with geometric facts and their uses. To teach convenient methods of using radicals in numerical computations.
12. To develop ability in solving quadratic equations by three processes—factoring, completing the square and the formula.

## PUPIL ACTIVITIES INTENDED TO REALIZE AIMS:

1. **To define algebra in the mind of the pupil as a useful, interesting and rational subject.**
   a. Bring out the fact that algebra is closely related to arithmetic, that where arithmetic is specific algebra is general. The gain in power which the general symbolic language of algebra affords over the particular numerical language of arithmetic constitutes its most important advantage.
   b. Solve a number of oral exercises and problems using symbols of operation, letters and equations. Stress the fact that a problem of some length translated into the shorthand of algebra becomes a short equation.
   c. A note book kept of the historical development of algebra with pictures and biographies of eminent mathematicians add interest and is of cultural value. The beginning can be made with the first use of symbols by Drophantus in the Fourth Century, A. D. Trace their refinement to present form.
   d. Review simple formulas of perimeters, areas, interest, etc., with which the pupil is familiar, and show their algebraic significance.
2. **To teach the solution of simple linear equations by means of a series of logical steps.**
   a. Learn and apply the four axioms relating to equations. A true equation will be true if the same number is added to both sides of the equation. Thus if
   $$n - 4 = 8, n - 4 + 4 = 8 + 4$$

A true equation will still be true if the same number is subtracted from both sides of the equation. Thus if

$$p + 2 = 5,\ p + 2 - 2 = 5 - 2.$$

A true equation will still be true if you multiply both sides of the equation by the same number. Thus if

$$\frac{x}{3} = 9,\ 3 \cdot \frac{x}{3} = 9 \cdot 3.$$

A true equation will still be true if you divide both sides of the equation by the same number. Thus if $3x = 36$

$$\frac{3x}{3} = \frac{36}{3}$$

3. **To lay the foundation for a sound method of problem solution.**
   a. Have the pupil master the steps of problem solution. First, read the problem carefully and list the quantities; second, represent each quantity with an abbreviation using one letter only; third, form an equation; fourth, solve the equation; fifth, check your answer to see if it meets the conditions of the problem.
      In forming the equation use:
      (1) Two abbreviations for the same quantity; or
      (2) Some fact stated or implied in the problem but not used in the list; or
      (3) Substitute in a formula.
      Use a great many oral problems from which to derive equations and use many simple equations and have them translated into verbal problems.
4. **To enlarge in the mind of the pupil the meaning of the minus sign as something more than a sign of operation.**
   a. A new use of the negative sign as a sign of character or quality must be clearly understood. Think of negative numbers as numbers less than zero—the idea of assets and liabilities is perhaps the best means of conveying the meaning of numbers less than zero. It is well, too, to introduce early in the course simple graphs which give the meaning of direction to signed numbers.
5. **To achieve mastery in the fundamental operations on signed numbers.**
   a. Give sufficient oral work until the idea of addition of algebraic quantities is mastered beginning with very simple monomial exercises and as mastery grows go on into more complex polynomials. Have the student thoroughly understand the meaning of co-efficients, exponents, powers, similar terms and rules governing operations with signed numbers. Check by reviewing the work step by step.
   b. Subtraction. In subtracting the change of signs in the subtrahend must always be done mentally, never in the written work. Check by difference + subtrahend = minuend.
   c. Multiplication. Review meanings of co-efficients and exponents. Learn rules of multiplication governing each. Arrange both multiplicand and multiplier according to the ascending or descending power of same letter. Learn the laws of signs in multiplication. Check by substitution of some small numerical value to each letter involved and find the corresponding numerical values of multiplier, multiplicand and product. The product of the numerical values of the multiplier and multiplicand should equal the numerical value of the product.
   d. Division. Through a number of model exercises on the board have the pupil deduce the rules governing the exponents and signs in division and why they are so. In division of polynomials the arrangement according to an ascending or descending power of some letter must be the same in both

dividend and divisor. Check by substitution of some small value for the letters involved or by multiplication.

e. In the note book have pictures and biographical sketches of John Wallis, Sir Wm. Rowan Hamilton.

6. **To develop power in the solution of equations and to make algebra a practical cultural subject rather than a mechanical drill. To strengthen the pupil's growing power to write the shorthand of algebra and to translate it into words.**

a. Charts and formulas develop power in solution of equations and make algebra more practical because better understood. Wells and Hart in **Modern First Year Algebra** use charts not only in solution of uniform motion problems, but also in age problems, mixture problems, coin problems, interest problems, digit problems, work problems, rectangle problems, etc. The charts help in reducing a problem to simple facts of the problem. Each answer should be checked by conditions of the problem to see if it be true.

b. Many problems may be reduced to the equation and solution not completed and the reverse process of using many equations and making verbal problems to fit the equation develop power in writing the shorthand of algebra and translating it into words.

c. The note book should be kept in good order and made as artistic as possible. Pictures of Sir Isaac Newton as a boy and as a man and a biographical sketch of him and his part in the development of mathematics and science should be added to it.

7. **To enlarge the pupils' control over multiplication and division.**

a. Mastery of factoring is so necessary as it simplifies all the future work in fractions. Products and corresponding cases of factoring are best taught in sequence. The following cases of factoring should receive emphasis:

| | |
|---|---|
| $ax + ay + bx + by$ | $x^2 + bx + c$ |
| $a^2 + 2ab + b^2$ | $ax^2 + bx + c$ |
| $a^2 - b^2$ | $a^3 + b^3$ |

(The sum and difference of two cubes is included because it is used in some of the standardized tests in algebra.)

b. Much of the work of special products and factoring should be done mentally. Only the results are written.

8. **To teach practical use of factoring in quadratic equations and problems.**

a. A large number of simple problems will develop more power in the use of factoring in the solution of equations than a few of more difficult ones. Variety of problems in the realm of the pupil's experience keeps up interest and applies the principles involved in more situations.

b. An original problem illustrated in pen and ink or color added to the note book adds interest. Be sure the problem is correct in statement and in solution before adding it.

9. **To give drill enough in fractions to make use of fractions in fractional equations possible. To develop the idea of function by ratio and proportion.**

a. A very thorough drill should be given in addition, subtraction, multiplication and division of fractions, using simple denominators which will give added drill in cases of factoring studied. Fractions with denominators the sum or difference of two cubes should be treated, as problems involving such fractions occur in standardized tests.

b. Ratio and proportion should be treated as types of fractions.

c The idea of function or of the relation of two quantities that change together can be given in simple form in treatment of ratio and proportion.

d. A picture of John Napier and a biographical sketch of his life may be added to the note book.

10. **To give in picture form the relation of quantities.**
    a. Through graphs the idea of function may further be developed and the meaning of the negative number as difference in direction be emphasized.
    b. Making and interpreting graphs of various kinds, solution of systems of equations by graphing are valuable drills.
    c. Graphs of the four different kinds should be made, viz: bar graph, broken line graph, curved line graph, and circle graph. Show that each of the four kinds is applicable to a particular kind of data.
        (1) Bar graphs are suitable for comparing different values when it is understood that one of the values shown has not changed or grown into the next value shown.
        (2) Broken line graphs may be used for the same kind of data as bar graphs, but they are particularly suitable for showing values of the same quantity at different times, when it is understood that one value shown has changed or grown into the next value shown.
        (3) Curved line graphs are used for the same purpose as broken line graphs. The difference is usually that between two values shown on a broken-line graphs the quantity may have experienced considerable fluctuation in value, while in a curved-line graph it is understood that the quantity has changed gradually from one value to the next without great fluctuations between these values.
        (4) Circle graphs are particularly suitable for showing how a certain quantity is divided into parts, that is, percentages.
        **Note:** The above has been taken from Algebra—Longley and Marsh.
    d. A very carefully drawn graph from original data makes a splendid addition to the note book.
    e. A picture and biographical sketch of Rene Descartes may be added to the note book at this point.

11. **To enlarge the pupil's acquaintance with geometric facts and their uses. To teach convenient methods of using radicals in numerical computations.**
    a. Simplification of radicals, addition and subtraction, multiplication and division of radicals must be thoroughly understood. The practical application to geometric problems enables the pupil to see at once the practical use of radicals.
    b. Pictures and biographical sketches of Pythagorus and Vieta may be added to the note book.

12. **To develop ability in solving quadratic equations by three processes: factoring, completing the square, and the formula.**
    a. Factoring quadratic equations when the factors are easily found may be reviewed.
    b. Solving quadratic equations by completing the square should be developed far enough to derive the formula for solving quadratic equations.
    c. Emphasis should be laid on the solution of quadratic equations by formula.

**TEXTBOOK (Optional):**

Rushmer and Dence—First Course in Algebra (American Book Co.).
(With Department's approval, any other text may be selected for the first year.)

**OTHER MATERIALS TO BE BE USED:**

Note book—Perry pictures of Eminent Mathematicians.
Graphing paper—Cross ruled black board.

## SUGGESTED ARRANGEMENT OF WORK BY SEMESTERS:

**First Semester.**

September: Use parts of introduction to introduce the work as needed.
Addition of positive and negative numbers.
Addition of monomials and polynomials.
Subtraction of positive and negative numbers.
Subtraction of monomials and polynomials.

October: Multiplication and division of positive and negative numbers.
Simple equations.
Identities and equations of condition.
Parenthesis.
Multiplication.

November: Parenthesis in equations.
Division.
Equations and problems.

December: Important special products and cases of factoring connected with the above.

January: Complete work with special products and factoring.
Solution of equations by factoring.

February: Fractions.
Equations containing fractions.

March: Review problems.
Graphs.
Linear systems.

April: Square root.
Radicals.

May: Quadratics solved by:
1. Review by factoring.
2. Completing the square.
3. Formula.

June: Ratio and proportion.
Examinations.

There should be short, diagnostic tests given once a week, more comprehensive tests once a month, quarterly department tests.

Standardized tests should be given at least twice a semester.

## BIBLIOGRAPHY FOR STUDENTS:

D. E. Smith—Number Stories of Long Ago (Ginn & Co.).
Fink—A Brief History of Mathematics (Open Coush Co.).
Susan Connington—Story of Arithmetic (Swan, Sonneschein & Co., London).

## BIBLIOGRAPHY FOR TEACHERS:

Wells and Hart—Modern First Year Algebra (D. C. Heath & Co.).
Edgerton & Carptenter—First Course in Algebra (Allyn & Bacon).
Longley & Marsh—Algebra (Macmillan).
Fine—Number System of Algebra (Ginn & Co.).
Cajori—A History of Mathematics (Macmillan).
Ball—A Short History of Mathematics (Macmillan).
Thorndike—Psychology of Algebra (Macmillan).
Smith—Teaching of Algebra (Ginn & Co.).
Paul Ligda—Teaching of Elementary Algebra (Houghton).
Symonds—Psychology of Errors in Algebra (Columbia U. Monograph).
Morrison—Principles of Teaching in Secondary Schools (University of Chicago Press).
Schultz—Teaching of Mathematics in Secondary Schools (Macmillan).
Mathematics Teacher (Magazine).
School Science and Mathematics (Magazine).
Evans—The Teaching of High School Mathematics (Houghton-Mifflin).

# ALGEBRA—ADVANCED

## AIMS OF INTERMEDIATE ALGEBRA:

1. **To review the principles of elementary algebra.**
2. **To enlarge the meaning and possibilities of algebraic expression.**
3. **To give pupils who do not go to college broader mathematical knowledge.**
4. **To prepare pupils for college by giving foundation work for higher mathematics which is a prerequisite to all engineering courses and research in physics and chemistry.**

## PUPIL ACTIVITIES INTENDED TO REALIZE AIMS:

1. **To review the principles of elementary algebra.**
   a. Review the fundamental operations.
   b. Use problems applicable to the life of the pupil.
   c. Review the types of factoring, stressing the remainder theorem, factor theorem and the sum and difference of two like powers.

2. **To enlarge the meaning and possibilities of algebraic expression.**
   a. Introduce the fractional negative and zero exponent and imaginaries using i for the unit.
   b. Study equations involving radicals, quadratics with fractional negative exponents and imaginary roots and equations in two and three unknowns.
   c. Bring out the functional idea in the work of variations in graphical interpretations and in the study of progressions.
   d. Explain logarithms, showing their practical application.
   e. Discuss determinants, permutations and combinations and elementary statistics.

3. **To give pupils who do not go to college broader mathematical knowledge.**
   a. Explain the use of progressions in computing compound interest and annuities.
   b. Give the pupils a workable knowledge of logarithms and statistics.
   c. Develop accuracy in the use of radicals and square roots.

4. **To prepare pupils for college by giving foundation work for higher mathematics which is a prerequisite to all engineering courses and research in physics and chemistry.**
   a. Develop accuracy and ease in the use of logarithms.
   b. Graph conic sections, familiarizing the student with the graph of the various systems of quadratic equations.
   c. Study limits and infinity.

## TEXTBOOK (Required):

Hawkes-Luby-Touton—New Second Course in Algebra (Ginn & Co.). Enlarged edition.

## OTHER MATERIALS TO BE USED:

Supplementary Texts—See Bibliography.
Compass and straight edge.
Solid figures to illustrate problems from solid geometry.

## SUGGESTED ARRANGEMENT OF WORK BY SEMESTERS:

**First Semester.**

1. Review of Fundamental Operations and Linear Equations, two days.
2. Factoring. Review cases taken in Algebra I and drill on the remainder theorem, factor theorem and the sum and difference of two like powers.
3. Fractions. Give a general review with special drill in addition and subtraction of fractions and the solution of fractional equations.
4. Linear Systems. Solve by graph, addition and subtraction and substitution with special emphasis on problems.
5. Exponents. The student should understand thoroughly the negative, fractional and zero exponent at end of first quarter.
6. Square Root. Review square root of numbers and then take the square root of algebraic expressions.
7. Radicals. Review simplification of simple radicals. Drill on rationalizing expressions, extracting the square root and factoring surds.
8. Functions and Their Graphs. The pupil should have a clear understanding of the meaning of **function.** Longley and Marsh, p. 237.
   Solve graphically linear, quadratic and cubic functions, stressing smoothness and accuracy of the figure.
9. Quadratic Equations. Solve by the four methods—factoring, graphing, completing the square and the formula, emphasizing the development of the formula. Solve the review exercises by the method best adopted to each.
10. Review semester's work. Examination.

**Second Semester.**

1. Imaginaries. Develop the four fundamental operations and solve equations with imaginary roots. Stress the use of i as the unit.
2. Irrational Equations. Show the importance of checking to find real roots.
3. Theory of Quadratic Equations. Dwell especially upon the relation between roots and coefficients as a means for a simple check. Study the character of the roots of an equation by means of the discriminant.
4. Graphs of Quadratic Equations. Show the relation of the equation and type of curve obtained. Solve systems of quadratics by means of the graph, finding the sets of roots common to the equations.
5. Systems Solvable by Quadratics. Make a study of the various types.
6. Progressions. Study arithmetical and geometric progressions and series. Show the application of progressions and series in computing compound interest and annuities. End of third quarter.
7. The Binomial Theorem. Expand and extract roots by use of the binomial theorem.
8. Ratio, proportion and variation.
9. Logarithms. Emphasize the fact that a logarithm is an exponent. Give drill in use of tables, interpolation and finding antilogarithms. Solve problems in multiplication, division, evolution and involution by means of logarithms. Give problems showing the practical application of logarithms.
10. Elementary Statistics. Study the fundamental devices used in making statistical analysis.

11. Determinants and Limits and Infinity. If time permits these chapters should be studied.
12. Review of semester's work. Examination.

**BIBLIOGRAPHY:**

Wells and Hart—Second Course in Algebra (Heath & Co.).
Edgerton and Carpenter—Intermediate Algebra (Allyn & Bacon).
Wentworth—Higher Algebra (Ginn & Co.).
Longley and Marsh—Second Course in Algebra (Macmillan).
Stone and Hart—Second Course in Algebra (Sanborn, N. Y.).
Wells—New Higher Algebra (Heath & Co.).
Ford and Ammerman—Second Course in Algebra (Macmillan).
Sykes & Comstock—Second Course in Algebra (Rand, McNally).

# BUSINESS ARITHMETIC

## AIMS OF BUSINESS ARITHMETIC:

1. **To train the pupil to perform accurately and easily the four fundamental operations with integers, decimals, common fractions and mixed numbers.**
2. **To develop in the pupil the habit of making neat figures and of arranging all numerical work in a form acceptable to business men.**
3. **To train the pupil in rapid mental work and in the more practical short methods of written work.**
4. **To train the pupil in problem reading, problem interpretation and problem solving.**
5. **To develop in the pupil an elementary knowledge of scale drawings and graphs.**
6. **To train the pupil to use equations and formulas as tools.**
7. **To train the pupil to estimate answers.**
8. **To train the pupil to check every result. This is done by the best workers in business offices.**

## PUPIL ACTIVITIES INTENDED TO REALIZE AIMS:

1. **To train the pupil to perform accurately and easily the four fundamental operations with integers, decimals, common fractions and mixed numbers.**

   (The major part of the work in business arithmetic must be done in class. Home work in this course is of little value. When working at home the pupil is inclined to take plenty of time, to use a pencil for work meant for mental drills, to do work carelessly, to use long processes where short ones are better, to use methods that counteract efforts made in class to develop mental alertness.)

   a. **Addition of Integers and Decimals.**

   (1) Most practice should be in vertical addition, but some attention should be given to horizontal addition as in pay rolls, sales sheets, statistical reports, etc.

   (2) In column addition pupils should be required to **think** 9, 15, 23, etc.

   (3) Drill in counting by 2's, 3's, 4's, 5's, 6's, 7's, 8's, 9's, 10's, 11's, 12's, until 100 is reached or passed.

   (4) Drill on short columns of figures until the result may be stated immediately.

   (5) In adding 59 to 635 mentally add 60 and subtract 1. The result is thus obtained with the least effort.

   (6) In adding 139 and 95 it is well to reverse the usual order and add from left to right, that is, $139 + 90 + 5$. This avoids carrying.

   (7) A great deal of timed drill work trains for accuracy, which is our prime object, and for speed, which is very desirable.

(8) Drill device: Take any two numbers as "a" and "b." Add the two numbers. Illustration,

| | |
|---|---|
| $a + b = c$ | $a = 32347$ |
| Add this sum to the number above | $b = 46943$ |
| $b + c = d$ | $c = 32347$ |
| Repeat | $d = 46943$ |
| $c + d = e$ | $e = 79290$ |

and so on until ten numbers are obtained (including the two numbers taken at first) and find the sum of all ten numbers. Check: The sum should be equal to 11 times the seventh number.

(9) Check addition by adding in the opposite direction, by casting out 9's or by casting out 11's. The last two are not absolute checks.

b. **Subtraction of Integers and Decimals.**

(1) Adopt a good method of subtraction and give much practice to insure facility in the use of the method adopted. The Austrian method is recommended. Give considerable practice in the Austrian method of making change.

(2) Drill orally in counting backwards by 2's, 3's, 4's, 5's, 6's, 7's, 8's, 9's, 10's, 11's, 12's, from 100, 99, 98, 97, 96, 95.

(3) Check subtraction by adding the remainder to the subtrahend to obtain the minuend.

c. **Multiplication of Integers and Decimals.**

(1) Special attention must be given to placing the decimal point in the product.

(2) Much practice should be given to multiplying integers and decimals by 10, 100, and 1000, and by 200, 3000, etc.

(3) The following aliquot parts should be mastered so that they become real tools and are used habitually:
Base 10; 2½, 1¼, 5, 7½, 3⅓, 6⅔.
Base 100; 20, 40, 60, 50, 25, 12½, 6¼, 37½, 62½, 87½, 33⅓, 66⅔, 16⅔, 8⅓, 110, 120, 90, 45.
Base 1000; 125, 250, 750, 375, 875, 900, 450, 1250, 1500.

(4) Computation may often be made easier by use of interchange principles, that is by interchanging the number of articles and the price of each article.

(5) Pupils should be trained to use aliquot parts of 60 and 6 for the rapid calculation of interest.
Base 60 days; 30, 20, 15, 12, 10, 5.
Base 6 days; 3, 2, 1.

(6) Short method of multiplying by 11.

(7) Approximation of the product by estimating the result.

(8) Practical short methods of multiplication should receive drill. This stimulates interest, produces real thinking and insures greater accuracy.

(9) Multiplication work may be checked by reviewing the work step by step or by dividing the product by one of the factors.

d. **Division of Integers and Decimals.**

(1) Special attention should be placed on locating the decimal in the quotient.

(2) Tests of divisibility by 2, 4, 8, 5, 10, 3, 6, 9, 12.

(3) Much practice in dividing integers and decimals by 10, 100, and 1000.

(4) Short method of dividing by 50, 25, 12, 5, 6¼, 250, 500, 33⅓.

(5) Check division by reviewing the work step by step or by multiplying the divisor by the quotient.

e. **Common Fractions and Mixed Numbers.**

(1) Since most fractions occuring in business arise from parts of the inch, foot, yard, pound, bushel and dollar, the major part of the drill given on fractions should involve only denominators common in these measures.

(2) Drill on rapid mental addition and subtracton of two common business fractions whose numerator is unity.

(3) Drill on rapid mental addition of any two small fractions whose common denominator is apparent.

(4) As a preparation for the four step process of multiplication, pupils should be required to find results like the following without the use of pencil.
⅔ of 14 = 8⅔. The pupil should think 28, 8⅔ ⅜ of 24 = 9. The pupil should think 3, 9. Give practice in determining it is preferable to divide, first by the denominator or multiply by the numerator.

(5) Drill in the "four step process" of multiplying mixed numbers.

(6) Drill in dividing a fraction by an integer to determine instantly which is better, to divide the numerator or multiply the denominator. The calculation should be done mentally and the result given at once.

(7) Choose one method for dividing when the divisor or dividend is a mixed number and give sufficient drill to make its use a tool.

(8) Practice in factors, common divisors, and common multiples should be limited to fractions with small denominators common to business.

At the close of the period of drill on the four fundamental operations and aliquot parts a standardized test gives a good measure of ability of the pupil to handle with ease and accuracy exercises involving these principles.

2. **To develop in the pupil the habit of making neat figures and of arranging all numerical work in a form acceptable to business men.**

a. Constant practice should be given in making neat figures and in writing numbers rapidly in good order for addition. Many errors in calculation are traceable to poorly made figures or careless column arrangement. Pupils should be required to take many addition exercises from dictation using plain and unit ruled paper.

Attention should be called to the use of capitals and hyphens in writing sums of money in words, as in a check; of acceptable forms of writing dollars and cents in figures in a check, etc.

3. **To train the pupil in rapid mental work and in the more practical short methods of written work.**

a. Constant practice throughout the course should be given in rapid mental work. The review drills at the end of each unit of work are excellent material on which to base the practice.

b. Care should be exercised in choice of short cuts—using only those whose practicability has been demonstrated. Sufficient drill should be given in each to make the pupil master of it.

4. **To train the pupil in problem reading, problem interpretation and problem solving.**

a. In the solution of problems the pupil has practical use of the mechanics of calculation. The teacher should give a large number of oral problems involving only very simple calculations so that the pupil will be able to focus his attention almost exclusively on the method of solution. Pupils who are good at the mechanics of calculation very often prove

weak in solving verbal problems because they do not understand the situation out of which the problem arises. The language used to state the problem is not fully comprehended. Pupils should be made thoroughly familiar with such expressions as:

½ as large as
⅓ larger than
sum of two numbers is
difference is
product is
decreased by ¼ of itself
increased by ⅓ of itself
diminished by ⅛ of itself
the quotient is
the product is

b. The pupil should be drilled in reading every problem very carefully to determine what is required to be found, what data are given from which this result is to be found, and how these data are to be used to yield the required result.

c. Checking of every answer by applying to the result the conditions of the original problem.

d. Percentage is the most important and the most difficult topic in business arithmetic. Thorough mastery of percentage is absolutely essential and the greater part of the year's work should center about it. A great number of simple problems realizes this aim than fewer difficult ones. The following topics should be mastered through abundant drill:

The expression of percents as common fractions and as decimals.

Aliquot parts commonly used in percentage,

Use of the decimal form of percent except in case of commonly used aliquot parts.

Exact meaning of:

½, .50, 50%
15.4%, .154
.2, 2%
⅛, ⅛%
2/10, n/30
8% more than
¾ as much as
⅓ less than
50% greater than
½ of 1%
⅛ of 1%
Money is worth 5%
Diminished by 2%
Increased by 10% of itself
Exceeded by 20%
Percent of increase over last year, etc.
Percent of decrease from last week's sales, etc.

e. Application of percentage to be thoroughly understood and computation of same to be mastered.

Billing and trade discount
Cash discount
Profit and loss. Stress basing profit on cost price
Manufacturing costs, profits, etc.
Marking goods
Commission and brokerage

f. Interest

Bank discount

g. Special topics

Taxes
Insurance
Investments, stocks, bonds, brokerage

5. **To develop in the pupil an elementary understanding of drawings and graphs.**

   a. Drawing the following forms of graphs, (1) bar graph to note quantities, (2) curve or broken line graph to depict fluctuations, (3) unit line graph (preferably vertical) to show percentage of component parts.

   b. Scales: meaning of ¼″=1′ etc.; drawing of simple floor plans to a given scale; reading of scale drawing by means of a ruler.

6. **To train the pupil to use equations and formulas as tools.**

   a. The use of formulas requires a simple knowledge of equations and of square root.

Simple formulas:

$$2\pi r = \text{Circumference}$$
$$\pi r^2 = \text{Area of circle}$$
$$2\ (l+w) = \text{Perimeter}$$
$$a\,b = \text{Area of rectangle}$$
$$Prt = \text{interest}$$
$$\frac{a\,b}{2} = \text{Area of triangle}$$

7. **To train the pupil to estimate answers.**

   a. Estimates are often made by inspection and in advance of careful accurate solution.
   Give practice problems in estimating amount of lumber needed to make certain shelves, pieces of furniture, a porch, a house, etc.
   Estimate costs of painting jobs.
   Estimate value of insured property destroyed by fire, as a basis of settlement with the insurance company.

8. **To train the pupil to check every result.**

   a. Pupils should be required to check every result before leaving it. Only work that is absolutely correct is of any value in business. The checking habit must be formed by continued practice.

## TEXTBOOK (REQUIRED)

Van Tuyl—New Essentials of Business Arithmetic (American Book Company)

## OTHER MATERIAL TO BE USED

Pay rolls, time sheets, sales sheets, sales tickets, cash records, order slips, invoices, inventories, tax returns sheets, insurance policy, checks, receipt forms, railway time table, annual reports of local industries may all be used if the teacher has a definite plan in mind.

"Rapid calculation pad" may be desirable for use a part of the time. But continued drill in dictated problems is the best drill.

Standardized tests are of great value. Kinney's Scale in Commercial Arithmetic, Test A, Parts I and II, are intended for use at the end of the first ten weeks. Test B is intended for use at the end of the first semester and Test C is devised for use at the end of the second semester.

## SUGGESTED ARRANGEMENT OF WORK BY SEMESTERS

### First Semester

September. Beginning of text to page 30 omitting page 11.
October. pp. 30-66 inclusive
November. pp. 67-99 inclusive
December. pp. 100-120
January. pp. 120-144. Omit paragraphs 267-299.

### Second Semester

February. pp. 145-165
March. pp. 165-190
April. pp. 191-218
May. pp. 219-266
June. pp. 267-290

## BIBLIOGRAPHY FOR TEACHERS

Campbell, M. M.: Workaday Arithmetic (Century Pub. Co., New York)
Finney and Brown: Modern Business Arithmetic (Henry Holt & Co., Chicago)
Thorndike, E. L.: The Psychology of Arithmetic (Macmillan Co.)
Thorndike, E. L.: The Thorndike Arithmetic (Rand MacNally Co., Chicago)

## BIBLIOGRAPHY FOR STUDENTS

Van Tuyl—New Essentials of Business Arithmetic (American Book Company)
D. E. Smith—Number Stories of Long Ago (Ginn & Co.)
Susan Connington—Story of Arithmetic (Swan, Sonneschein & Co., London)
The Practice of Teaching in Secondary Schools (University of Chicago Press)

# GEOMETRY—PLANE

## AIMS OF PLANE GEOMETRY:

1. **To develop the ability for concise and logical thinking.**
2. **To awaken an appreciation of geometry.**
3. **To acquire ideas and concepts used by the world in its quantitative thinking.**
4. **To grasp a perception of space.**
5. **To increase the ability to understand science and to investigate its undeveloped fields.**

## PUPIL ACTIVITIES INTENDED TO REALIZE AIMS:

1. **To develop the ability for concise and logical thinking.**
   a. Give a little practice in syllogisms from everyday life to impress on the class the general form of a demonstration.
   b. Use accurate constructions for all drawings by means of a straight edge, compass, protractor and colored pencils.
   c. Generalize all figures.
   d. Adopt one form of writing proofs. Number the steps in the proof and calculate acceptable reasons in parallel columns.
   e. Give all authorities by complete statements. Discourage such purely local symbols as s a s and other similar symbols.
   f. Require a formal demonstration for all theorems and exercises.
2. **To awaken an appreciation of geometry.**
   a. Have the students keep a notebook which will include pictures and short biographies of noted mathematicians; photographs showing the use of geometry in engineering achievements and geometric designs used for decorative purposes.
   b. Show the practical application of geometry and its importance in art, applied science, engineering, and commerce.
   c. Form a mathematics club as an extra-curricula activity.
   d. Suggest to the pupils the fact that geometry aids citizenship in that it familiarizes them with the technique of setting up a valid proof and makes them appreciate the place of geometry in the development of civilization.
3. **To acquire ideas and concepts used by the world in its quantitative thinking.**
   a. Study the mensuration of triangles, polygons, circles, sectors, and segments.
   b. Develop the concept of ratio and proportion.
   c. Introduce trigonometric functions.
   d. Demonstrate the use of a pantograph, Gunther's scale and Peaucelliar's linkage.
   e. Have students construct a pantograph, 30-60 degree and 45-45 degree right triangles and a protractor which might be used in field or board work.
4. **To grasp a perception of space.**
   a. Study such fundamental ideas as congruence, similarity and symmetry with practical applications.
   b. Emphasize the use of the locus in the solution of problems.

5. **To increase the ability to understand science and to investigate its undeveloped fields.**
   a. Do as much field work as possible by measuring buildings, trees and flag poles on the campus. Draw an imaginary river or lake on the campus and form problems involving congruent or similar triangles to compute its width.
   b. Demonstrate by laboratory method the application of geometry in regard to pulleys, cog-wheels, etc.

## TEXTBOOK (REQUIRED)

Hawkes, Luby, Touton—Plane Geometry (Ginn & Co.)

## OTHER MATERIALS TO BE USED

Supplementary Texts and other books—See bibliography.

Pictures—Famous mathematicians and decorative geometric designs.

Instruments—Blackboard compasses, triangles, parallel ruler, transit, carpenter's squares, colored crayons, pointers, yard and meter sticks, large protractor, fifty-foot tape, pulleys and pantograph.

## SUGGESTED ARRANGEMENT OF WORK BY SEMESTERS

### First Semester

1. Inspirational unit.
   Give a short history of geometry and a brief resumé of the practical applications of geometry by bringing out its importance in life and industry.
2. Unit on Terms, Fundamental Principles and First Theorems.
   Study by experiment, drawing, computation and informal discussion on the fundamental principles of geometry. Discuss the types of angles and such axioms and postulates as are needed. Teach a few simple constructions.
   Theorem 9 Familiarize the student with a formal proof.
3. Unit on Triangles.
   a. Congruence of triangles in general. Theorems 1, 3, 13. Study theorem 4.
   b. Right triangles congruence—theorems, 8, 19, 20; other —theorems 24, 25.
   c. Isosceles and equilateral triangles. Theorems 2, 21, 22, 23.
   d. Exercises—75 per cent should be solved.
4. Unit on Parallel Lines.
   a. Conditions under which lines are parallel. Theorems 5, 11, 34, exercises 21, 22, 23, 24.
   b. Facts about parallel lines. Theorems 7, 6, 10, 12, and corollaries, exercises 14, 15, 16, 17.
   c. Exercises. Solve 75 per cent.
5. Unit on Parallelograms.
   a. Definition and construction of a parallelogram.
   b. Conditions under which a quadrilateral is a parallelogram. Theorems 15, 16, 18.
   c. Facts about a parallelogram. Theorem 14 and corollary 17. Exercises 55, 56, 57, 58, 59.
   d. Special parallelograms. Definition and construction of a rectangle, square and rhombus. Theorems 26, 27, and corollaries.
   e. Exercises. Solve 75 per cent.

6. Unit on Midperpendiculars and Angle Bisectors.
   a. Midperpendiculars. Theorems 28, 29, and corollary 118. Constructions 3 and 4, pp. 134-5.
   b. Angle bisectors. Theorems 30, 31. Constructions 5 and 6, pp. 136-137.
   c. Introduce the idea of loci. Use other text as Palmer-Taylor-Farnum or Bush and Clark.
   d. Exercises. Solve 75 per cent.
7. Unit on Polygons and Other Topics.
   a. Sum of angles of a polygon. Theorems 32, 33.
   b. Other topics. Theorems 35, 36, 37, and corollaries.
   c. Exercises. Solve 75 per cent.
8. Unit on Inequalities.
   a. Axioms, pp. 72, 73. Theorem 38.
   b. Theorems 39, 40, 41, 42, 43.
   c. Exercises. Solve 75 per cent.
9. Review of First Quarter's Work. Examination.

Book II. The Circle.

1. Unit on Chords, Arcs and Central Angles, pp. 89-102.
   a. Definitions. Theorems 1-11.
   b. Exercises. Solve 75 per cent.
2. Unit on Tangents and Secants, pp. 103-115.
   a. Construction of tangent to a circle at a given point.
   b. Theorems 12-17.
   c. Exercises. Solve 75 per cent.
3. Unit on Angle Measurement.
   a. Discuss types of angles found in circles.
      Theorems 18-21.
      Theorem 23 to be discussed.
   b. Exercises—solve all and give supplementary problems—Palmer-Taylor-Farnum—185-189.
4. Unit on Construction.
   a. Constructions 1-9; exercises 121, 126, 127, 131, 134, 135.
   b. Theorem 22, p. 124.
5. Unit on Loci.
   a. Simplify the solution of problems through the use of loci. pp. 144-154.
   b. Loci expressing one condition.
   c. The Intersection of loci.
   d. Exercises from supplementary texts.
   f. Review of semester's work. Examination.

## SECOND SEMESTER

Book III

Ratio, Proportion and Similar Figures

1. Unit on Principles of Proportion.
   a. Mechanical manipulation of ratios and proportions pp. 155-159, 161, 164, 191, 203.
2. Unit on Similar Triangles.
   a. Conditions under which lines are proportional.
      Discuss theorems 1 and 2 and corollary 260.
      Theorems 3, 4, 5, 9.
   b. Similar triangles. Theorems 6, 7, 8.
   c. Introduce trigonometric ratios. pp. 187-191.
      Give practical problems.
   d. Exercises. Solve 75 per cent.

3. Unit on Miscellaneous Topics.
   a. Theorems included in pp. 192-204. Discuss theorem 20, p. 202.
   b. Exercises. Solve 75 per cent.
4. Unit on Field Work.
   a. Interest the pupil by making his work concrete and show the practical side of geometry. pp. 205-207. Palmer-Taylor-Farnum pp. 280-286.
   b. Use a plane table if possible.
5. Unit on Construction.
   a. Constructions pp. 210-213.
   b. Exercises. Solve 50 per cent.
6. End of third quarter's work. Examination.

Book IV

Surface Measurement. Areas.

1. Unit on Areas.
   a. Theorems on surface measurement pp. 214-223.
   b. Solve 75 per cent.
2. Unit on Ratio of Areas of Polygons.
   a. Relationship of the areas of Polygons in terms of their dimensions pp. 224-228.
   b. Exercises. Solve 50 per cent.
3. Unit on Projections.
   a. Theorems and exercises pp. 229-242.
4. Unit on Construction.
   a. Further understanding of the principles of areas through concrete work pp. 243-248.

Book V

Regular Polygons and the Measurement of the Circle.

1. Unit on Regular Polygons.
   a. Relationship of a regular polygon to a circle. pp. 260-263. Theorems 1, 2, 3-6.
   b. Exercises 32-38.
2. Unit on Construction of Regular Polygons.
   a. Inscribed and circumscribed circles of a regular polygon. pp. 249-259.
   b. Exercises. Solve 75 per cent.
3. Unit on the Similarity of Regular Polygons.
   a. Similarity of regular Polygons pp. 264-265.
   b. Study the computation of pi—pp. 272-273.
4. Unit on the Measurement of Arcs, Sectors and Segments.
   a. Theorems 8, 9, 10, 11.
   b. Exercises. Solve 75 per cent.
5. Review of year's work. Examination.

## BIBLIOGRAPHY


Wells & Hart—Plane Geometry. (D. C. Heath & Co.)

Palmer-Taylor-Farnum—Plane Geometry. (U. of Chicago Press)

Avery—Plane Geometry. (Allyn & Bacon)

Schultze—Teaching of Mathematics in Secondary Schools. (Macmillan & Co.)

Evans—The Teaching of High School Mathematics. (Houghton Mifflin Co.)

The Reorganization of Mathematics in Secondary Education. (The Mathematical Association of America, 1923)

Sykes—A Source Book of Problems for Geometry. (Allyn & Bacon)

Klein—Famous Problems of Elementary Geometry. (Ginn)

Smith—Euclid, His Life and System. (Ginn)
Manning—Non-Euclidean Geometry. (Ginn)
Ball—A Short History of Mathematics. (Macmillan & Co.)
Cajori—A History of Mathematics. (Macmillan & Co.)
Gow—A Short History of Greek Mathematics. (Stechert)
Ball—Mathematical Recreations. (Macmillan Co.)
Ford and Ammerman—Plane Geometry. (Macmillan)
Smith—The Teaching of Geometry. (Ginn & Co.)
Young—The Teaching of Mathematics. (Longmans, Green)
Seymour—Plane Geometry. (American Book Co.)
Smith, R. R.—Beginner's Geometry. (Macmillan)
Rupert—Famous Geometrical Theorems and Problems. (Heath & Co.)
Stamper—History of Teaching Elementary Geometry. (Teachers' College, Columbia U.)
Mathematics Teacher—Lancaster, Pa.
School Science and Mathematics—Chicago, Ill.
Minnick—Geometry Tests—(School Publishing Co.)
Schorling: New Type of Geometry Examination—(415 West 123rd St., New York, N. Y.)
Roger: Tests of Mathematical Ability and Their Projective Value (Tests)
The Problem of Mathematics in Secondary Education—(Government Publishing Office, 1920)
The National Com. of Math. Requirements.
The Reorganization of Mathematics in Secondary Education
The Math. Association of America Inc., Lancaster, Pa. Feb. 1923

# GEOMETRY—SOLID

## AIMS OF SOLID GEOMETRY:

1. To lay a foundation for future work by teaching the terminology of solid geometry, by teaching the methods of passing planes, proving lines and planes parallel, lines and planes perpendicular and dihedral angles equal. To give an understanding of line projections and skew lines.
2. To teach the methods of finding the volumes and parts of prisms, cylinders, cones and pyramids.
3. To teach the properties of polyhedrons, their construction and the method of determining them through their angles.
4. To teach the method of finding the volume of symmetric and similar tetrahedrons and of prismatoids.
5. To teach the general properties of the sphere.
6. To teach the method of finding the area and volume of the sphere and its different parts.
7. To teach the properties of spherical triangles and of their practical applications.
8. To develop the scientific imagination of the pupil.

## PUPIL ACTIVITIES INTENDED TO REALIZE AIMS:

1. To lay a foundation for future work by learning the terminology of solid geometry, and by teaching the methods of passing planes, proving lines and planes parallel, lines and planes perpendicular and dihedral angles equal. To give an understanding of line projections and skew lines.
   a. Two class periods should be devoted to a review of definitions and the more important theorems of plane geometry which carry over into solid geometry. The first few pages of the solid geometry text is usually given over to such definitions and statements of theorems.
   b. Making of simple solid geometry figures by the pupils from light cardboard helps the pupil to "see the figure." Stereoscope cards of solid geometry figures is also a great help in getting the spatial idea.
   c. Formal proof of theorems 1-18 with sufficient, well chosen exercises develops skill in handling solid geometry work. Use carefully the queries scattered throughout the book to stimulate discussion and to arouse the pupils "to regard the subject not merely as a logical sequence of theorems but as a subject inviting reflection and the play of speculation."
   d. Formal proof of theorems 19-26, omitting proof of theorem 25 with queries and exercises.
   e. Formal proof of theorems 27-28 with queries and exercises.
   f. Study construction 5 with corollary.
2. To teach the methods of finding the volumes and parts of prisms, cylinders, cones and pyramids.
   a. A clear understanding of definitions and formal proof of theorem 1 in Book VII is necessary as a starting point to realize aim 2.

b. Formal proof of theorems bearing on volumes of prisms, cylinders, cones, and pyramids should be mastered as they appear in the course.
c. Watch carefully, in assigning exercises, the time actually required for their completion especially in those involving computation of length without much reasoning.

3. **To teach the properties of polyhedrons, their construction and the method of determining them through their angles.**
   a. Formal proof of theorems bearing on properties of polyhedrons should be mastered as they appear in the course.
   b. Constructions 1-3 should be carefully done.
   c. Three of the "five regular bodies" should be made by each pupil. Light weight cardboard and passepartout binding are very easy to work with.

4. **To teach the method of finding the volume of symmetric and similar tetrahedrons and of prismatoids.**
   a. Formal proof of theorems 33, 34, 35, and 36, should be carefully performed. Theorem 37 should be studied but pupil not held responsible for the proof.
   b. All definitions and statements should be learned.
   c. Sufficient well chosen exercises should be solved, making practical application of principles learned.
   d. Of the review exercises sufficient number should be solved to fix principles in mind.

5. **To teach the properties of the sphere.**
   a. A clear understanding of definitions of the sphere and spherical properties is necessary.
   b. Formal proof of those theorems dealing with spherical properties followed by queries and exercises.

6. **To teach the method of finding area and volume of the sphere and its different parts.**
   a. Formal proof of those theorems dealing with area and volume of spheres.
   b. Careful choice of exercises dealing with arithmetical computation.
   c. Discussion of queries following theorems.

7. **To teach the properties of spherical triangles and of their practical applications.**
   a. Formal proof of those theorems dealing with spherical triangles.
   b. Solution of exercises which emphasize practical application.
   c. Discussion of queries following theorems.

8. **To develop scientific imagination of the pupil.**
   a. Reserve time each day for discussing the queries which follow the greater portion of the theorems.

**TEXTBOOK** (Required):

Hawkes, Luby and Touton—Solid Geometry. (Ginn & Co.)

**OTHER MATERIAL TO BE USED:**

Board compasses.
Rulers.
Light weight cardboard.
Models for use in Solid Geometry.
Stereoscope and slides of Solid Geometry figures.
Set of mechano.
Knitting needles.
Five wire needles of different length.
Umbrella ribs also make good material with which to construct figures.

## DISTRIBUTION OF TIME IN SOLID GEOMETRY:

**Book VI—Five weeks.**

Discuss the major portion of the queries and, if possible, solve fifty per cent of the exercises; covering 28 theorems, 5 constructions, 15 corollaries, 130 queries and as many well chosen exercises as time permits.

**Book VII—Eight weeks.**

Cover 37 theorems, 3 constructions, 24 assumptions and corollaries, 94 queries and as many well chosen exercises as time permits.

**Book VIII—Six weeks.**

Cover 20 theorems, 20 corollaries and special cases, 72 queries and as many well chosen exercises as time permits.

## BIBLIOGRAPHY FOR STUDENTS:

Cajori—A History of Mathematics (Macmillan Co.).
Popular Science Monthly.
Scientific American.
Popular Mechanics.

## BIBLIOGRAPHY FOR TEACHERS:

Schultze—Teaching of Mathematics in Secondary Schools (Macmillan Co.).
Morrison—The Practice of Teaching in Secondary Schools (University of Chicago Press).
Cajori—History of Mathematics (Macmillan Co.).
Mathematics Teacher.
School Science and Mathematics.

# HIGH SCHOOL MATHEMATICS (SPECIAL)

## AIMS OF HIGH SCHOOL MATHEMATICS:

1. **To show the utility of elementary algebra. To teach the solution of simple linear equations by means of a series of logical steps. To lay the foundation for a sound method of problem solution. To define algebra in the mind of the pupil as a useful, interesting, and rational subject.**
2. **To teach the simple uses of proportion, and other easy fractional equations, together with the necessary multiplication of monomial factors; and meanwhile to keep algebra in the realm of things understood rather than in the realm of mechanical rule.**
3. **To illustrate the functional relation as expressed by words, tables, graphs, and equations. To enlarge in the mind of the pupil the meaning of the minus sign and to show how the addition, subtraction, multiplication and division of signed numbers can be understood and explained. To teach the solutions and uses of linear equations in two unknowns.**
4. **To teach the solution of quadratic equations by factoring and by a formula. To enlarge the pupil's control over the four processes. To strengthen the pupil's growing ability to write the shorthand of algebra and translate it into words.**
5. **To provide the pupil with much interesting material with which to test his skill and understanding. To review the most useful parts of elementary algebra.**

**Note:** This course is designed for the pupil who has trouble with mathematical processes. Only minimum essentials of algebra are offered with paralleled arithmetical computations. These aims are taken from Barber's Everyday Algebra table of contents.

## PUPIL ACTIVITIES INTENDED TO REALIZE AIMS:

1. **To show the utility of elementary algebra. To teach the solution of simple linear equations by means of a series of logical steps. To lay the foundation for a sound method of problem solution. To define algebra, in the mind of the pupil as a useful interesting and rational subject.**
   a. Review the formulas with which the child is familiar emphasizing the fact that algebra is a written language or shorthand in which the sentences are called formulas or equations.
   b. Solve a number of simple problems involving these formulas emphasizing that all quantities used in one formula must be in the same units of measure.
   c. Give a number of problems from which the pupil may derive his own formulas.
   d. Stress the five steps of solving a problem which involves a formula.
      (1) Select the correct formula.
      (2) Substitute in the formula the numbers given in the problem.
      (3) Tell what question the equation suggests.
      (4) Solve the equation.
      (5) Check the answer.

(Tests given in the text at the close of each unit of work are excellent for diagnostic purposes).

e. Problem solving is done by a series of steps:
   (1) Read the problem carefully.
   (2) List the quantities mentioned.
   (3) Represent each quantity by an abbreviation using one only.
   (4) Make an equation.
   (5) Solve the equation.

   Give a number of simple problems for the pupils to solve orally.

f. Teach simplification of equations by subtraction and by addition.

g. Analyze a quantity of problems of wide variety and solve many orally, some written.

h. Teach the use of parenthesis.

**Note:** Barber lays great emphasis on problem solving. Very little mechanical drill is given.

2. **To teach the simple uses of proportion and other easy fractional equations, together with the necessary multiplication of monomial factors; and meanwhile to keep algebra in the realm of things understood rather than in the realm of mechanical rule.**

a. Review operations on fractions in arithmetic. Review the fact that a true equation will still be true if you multiply both sides of the equation by the same number. Be sure to multiply every term. The following is a good device:

$$\frac{x}{3}+\frac{x}{4}+\frac{x}{2}=143$$

$$12\cdot\frac{x}{3}+12\cdot\frac{x}{4}+12\cdot\frac{x}{2}=143\cdot 12.$$

Simplify a number of equations by multiplication.

b. Give a number of problems involving areas, volumes, angles, uniform motion.

c. Simple fractional equations involving both multiplication and subtraction.

d. Do a number of scale drawings.

3. **To illustrate the functional relation as expressed by words, tables, graphs, and equations. To enlarge in the mind of the pupil the meaning of the minus sign and to show how the addition, subtraction, multiplication and division of signed numbers can be understood and explained. To teach the solutions and uses of linear equations in two unknowns.**

a. Construct graphs of temperatures of changing centimeters to inches, rates of trains in miles per hour.

b. Explain graphs as a bridge from algebra to geometry and the new meaning of the negative number—that is of direction.

c. Teach the solution of pairs of linear equations by graphing and by elimination.

d. Drill on addition, subtraction, multiplication and division of algebraic quantities.

e. Drill on the three signs of a fraction.

4. **To teach the solution of quadratic equations by factoring and by a formula. To enlarge the pupil's control over the four processes. to strengthen the pupil's growing ability to write the shorthand of algebra and translate it into words.**

a. Teach special products and cases of factoring pertaining to such products. Stress the fact that factoring is the reverse process of multiplication.

b. Give the formula for solving a quadratic equation of the form $ax^2+bx+c=0$ and give enough drill to fix the method in mind.

c. Solve problems in which the equations are solved by factoring.

d. Give drill in long division relating the work to multiplication.

5. **To provide the pupil with much interesting material with which to test his skill and understanding. To review the most useful parts of elementary algebra.**

a. Review the facts that formulas and equations are the most important and most useful parts of algebra and why they are.

b. Review the processes used in solution of formulas and equations viz.: Substitution, addition, subtraction, multiplication and division.

c. Review all the algebraic symbols.

d. Review solution of problems.

e. Give quantities of problems and exercises taken from the realm of the pupil's experience. Encourage the "making up" of problems by the pupils.

f. Contests in drawing graphs and of wording, solving and illustrating by pen and ink drawing original problems add interest for the pupil.

**TEXTBOOK** (Optional)

Barber: Everyday Algebra (Houghton Mifflin Co.)

**OTHER MATERIAL TO BE USED:**

Graphing paper
Colored crayons
Cross ruled black board

**BIBLIOGRAPHY FOR STUDENTS:**

Popular Mechanics.
Popular Science Monthly.
School Science and Mathematics.

**BIBLIOGRAPHY FOR TEACHERS:**

Edgerton and Carpenter—First Course in Algebra (Allyn & Bacon)
Mabel Sykes—First Course in Algebra (Houghton Mifflin)
Evans—Teaching of High School Mathematics (Rand MacNally)
Morrison—Principles of Teaching in Secondary Schools (University of Chicago Press)

# TRIGONOMETRY

## AIMS OF TRIGONOMETRY:

1. To develop the trigonometric functions of acute angles.
2. To develop the use of the table of natural functions.
3. To establish the use of logarithms and to develop facility in the use of logarithmic tables of five places.
4. To develop the practicability of logarithms in solving trigonometry problems dealing with the right triangle.
5. To develop the trigonometric function of any angle.
6. To develop the functions of the sum or the difference of two angles.
7. To develop the trignometric solution of the oblique triangle.
8. To develop the miscellaneous applications of trigonometry.
9. To develop the value of graphs of functions.
10. To develop an acquaintance with trigonometric identities and equations.
11. To develop the applications of trigonometry to algebra.

## PUPIL ACTIVITIES INTENDED TO REALIZE AIMS:

1. To develop the trigonometric function of acute angles.
   a. By use of similar right triangles develop the fact that the ratios of their corresponding sides, pair by pair are equal and and remain unchanged as long as the acute angle A at the vertex remains unchanged.
   b. Develop the fact that for every value of an acute angle A there are certain numbers that express the values of the ratios of the sides in all right triangles that have this acute angle A.
   c. Develop the fact that there are six of these ratios and have the pupil learn these six ratios in formula form and in terms of the sides of the triangle.
2. To develop the use of the table of natural functions.
   a. Study carefully the practical applications of the natural functions to a right angled triangle in the following cases:
      (1) Acute angle and hypotenuse.
      (2) Acute angle and opposite side.
      (3) Acute angle and adjacent side.
      (4) Hypotenuse and side.
      (5) Two sides.
   b. Solve problems dealing with each application.
3. To establish the use of logarithms and to develop facility in the use of logarithmic tables of five places.
   a. Extend the knowledge of logarithms gained in algebra through a study of the history of logarithms.
   b. Solve problems dealing with right triangles in which the process of interpolation is necessary.
   c. Solve problems dealing with whole numbers, integer and decimal, and decimals for the purpose of establishing the relationship between characteristic and mantissa.

d. Solve problems dealing with the logarithm of a product, of a quotient and of a power.

4. **To develop the practicability of logarithms in solving trigonometry problems dealing with the right triangle.**

   a. Stress the fact that logarithms are not a necessity but a very great convenience in shortening computations.

   b. Solve problems of right trianglse using logarithms dealing with the following cases:

      (1) Acute angle and the hypotenuse.
      (2) Acute angle and the opposite side.
      (3) Acute angle and the adjacent side.
      (4) Hypotenuse and side.
      (5) Two sides.

   c. Solve problems dealing with areas of isosceles triangles and regular polygons.

5. **To develop the trigonometric functions of any angle.**

   a. Extend the application of trigonometry to any triangle. Enlarge the pupil's idea of angles to include negative angles and angles greater than 360°.

   b. Plot various angles, and the solution of problems.

   c. Develop the line value of functions, and the idea of algebraic signs of trigonometric functions being determined by the quadrant in which the generator lies.

   d. Develop the idea of reduction of functions to the first quadrant.

   e. Develop the idea of the functions of negative angles.

   f. Develop the relations which exist between the functions of any angle.

6. **To develop the functions of the sum or the difference of two angles.**

   a. Develop the formula for the sine and cosine of the sum of two angles using the geometric figure and algebraic equations.

   b. All major formulas should be carefully committed to memory by the pupil.

   c. The formula for the tangent and cotangent of the sum of two angles should be developed, by the pupil, from the relationship existing between those functions and the sine and cosine.

   d. The formula for the sine and cosine of the difference of two angles should be developed by the pupil using the geometric figure and the algebraic equations.

   e. The formula for the tangent and cotangent of the difference of two angles should be developed by the pupil from the relationship existing between those functions and the sine and cosine.

   f. Problems should be solved using the formulas for the functions of the sum or the difference of two angles.

   g. Develop the formulas for the functions of twice an angle from the formulas for the sum of two angles.

   h. Develop the formulas for the functions of half an angle from the Pythagorean formula nivolving sine and cosine.

   i. Develop the formulas for the sums and differences of functions.

   j. A number of exercises should be solved involving the formulas the pupil has just developed and committed to memory.

7. **To develop the trigonometric solution of the oblique triangle.**
   a. Review certain geometric properties of a triangle.
   b. Develop the Law of Sines using the geometric figure and extend the Law of Sines with respect to the diameter of the circle circumscribed about a triangle.
   c. Solve problems making application of the Law of Sines.
   d. Develop the ambiguous case and determination of the number of solutions to be expected.
   e. Develop the Law of Cosines using the geometric figure.
   f. Solve problems using the Law of Cosines.
   g. Develop the Law of Tangents using Law of Sines and the Theory of Proportion.
   h. Solve exercises using the Law of Tangents.
   i. Develop the application of these laws to the solution of the oblique triangle.
   j. Develop the possibility of finding the angles of an oblique triangle, by the Law of Cosines if three sides are given.
   k. Develop the formulas for finding the area of a triangle when the following parts are known:
      (1) Two sides and the included angle.
      (2) Two angles and any side.
      (3) The three sides.
   l. Solve exercises and problems using the formulas for finding the area of a triangle.

8. **To develop the miscellaneous applications of trigonometry.**
   a. The pupil should solve problems involving the right triangle, the oblique triangle, areas, field measurements, surveying.

9. **To develop the value of graphs of functions.**
   a. Pupil should become familiar with circular measurements, radian, relation of radian to degree measure.
   b. Pupil should become acquainted with functions of small angles.
   c. Pupil should graph a function, a quadratic function, a sine, a cosine, tangent and cotangent.

10. **To develop an acquaintance with trigonometric indentities and equations.**
   a. Pupil should be able to differentiate between an equation and an identity.
   b. Prove a number of identities, choosing those to be proved carefully so as to use all the formulas rather than repetition of a few.
   c. Study carefully the solution of a trigonometric equation.
   d. Solve a number of trigonometric equations and systems of equations.

11. **To develop the applications of trigonometry to algebra.**
   a. Pupil should become familiar with:
      (1) De Moivre's Theorem.
      (2) De Moivre's Theorem, extended.
      (3) Roots of Unity.
      (4) Roots of Numbers.
      (5) Properties of Logarithms.
      (6) Exponential Series.
      (7) Euler's Formula.

**TEXTBOOK** (Required)

Wentworth-Smith—Plane Trigonometry and Tables (Ginn & Co.).

## OTHER MATERIALS TO BE USED:

Blackboard compass.
Blackboard protractor.
Colored chalk.
Engineer's transit.
Plane table.
Slide Rule.

## SUGGESTED ARRANGEMENT OF WORK:

| | |
|---|---|
| Function of Acute Angles | 2 weeks. |
| Use of Natural Functions | 1 week. |
| Introduction to tables | 2 weeks |
| Logarithms | 2 weeks. |
| Right Triangles and Logarithms | 2 weeks. |
| Functions of Any Angle | 2 weeks. |
| Sum or Difference of Two Angles | 2 weeks. |
| Oblique Triangles | 3 weeks. |
| Miscellaneous Applications | 2 weeks. |
| Application to Algebra | 1 week. |

**Note:** To complete Wentworth-Smith's Plane Trigonometry in one semester is very much of an undertaking, hence only the fundamental parts should be stressed. It is more important that the pupil be master of fundamental principles than that the entire book be completed. It has been suggested that chapters IX, X, and XI be omitted.

## BIBLIOGRAPHY FOR TEACHERS:


Florian Cajori—A History of Mathematics (Macmillan Co.).
Florian Cajori—History of Elementary Mathematics (Macmillan Co.).
David E. Smith—History of Modern Mathematics (John Wiley & Son).
Merriman Mansfield—The Solution of Equations (John Wiley & Son).
Horace W. Marsh—Constructive Text Book of Practical Mathematics (John Wiley & Son).
Robert E. Moritz—Elements of Plane Trigonometry (High School Edition) (John Wiley & Son).
School Science and Mathematics (Turton & Warner, Publishers, Mount Morris, Illinois).


## BIBLIOGRAPHY FOR PUPILS:


Trade magazines.
Scientific American.

# Notes and Criticisms

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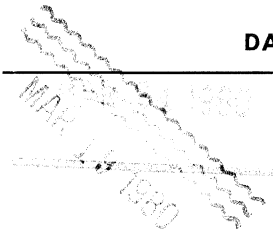

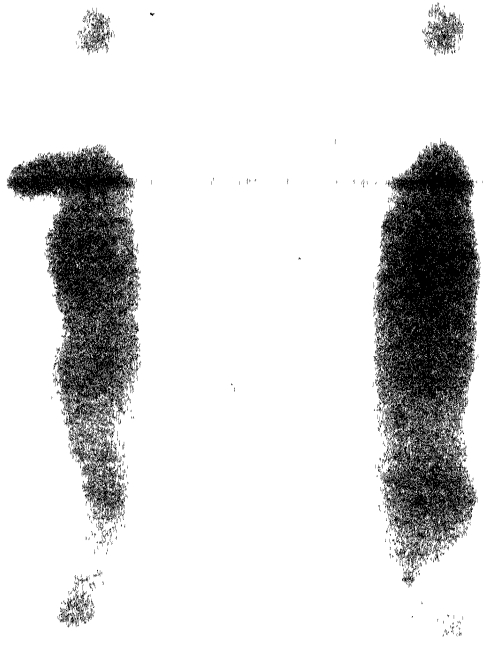

HONOLULU STAR-BULLETIN, LTD.